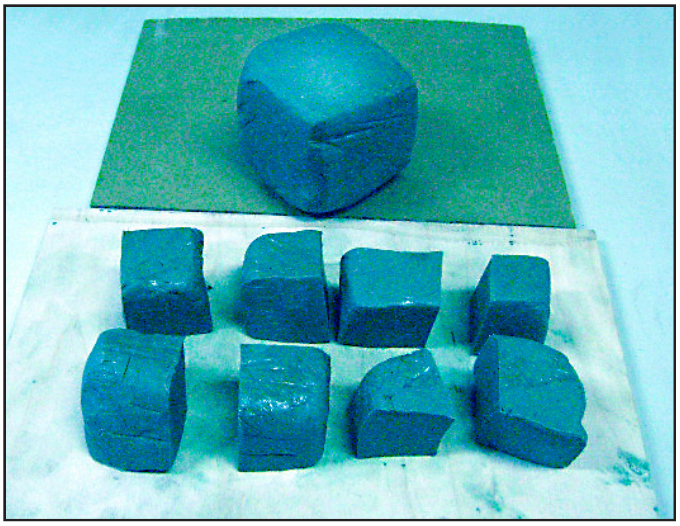
Shrinking Cubes

Discover how area and volume change as you scale an object down.



Materials

(Enough for two people working together.)

- 8 oz. or more of plasticine (plastic modeling “clay”); a large amount is easiest to work with
- old newspapers
- dinner knife or steel ruler for cutting the plasticine
- small block of wood or particleboard (about 3″ × 5″ or slightly larger) for shaping the cubes
- blank paper (1 or 2 sheets)

Tips

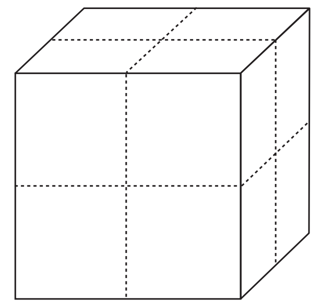
- Plasticine leaves an oily residue, so cover your work surface with newspaper or other paper to protect it.
- Plasticine can be difficult to cut. Using a rocking motion with your knife or ruler makes it easier.

To Do and Notice

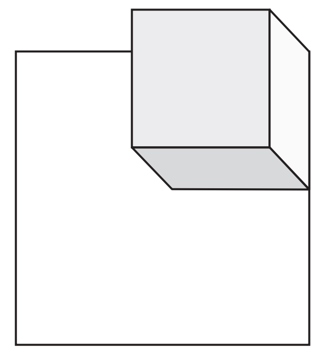
1. Shape the plasticine into a cube by turning it on various sides and using the block of wood to push or pound it. (You won’t make a perfect cube this way, but get it as close as you can.)

2. Outline the base of your cube on paper. This will help you compare your little cubes to your original cube.

3. If you cut your cube so that you have a number of cubes that are half the length of your original cube per side, how many little cubes do you think you'll have? Cut up your cube as shown in the illustration and see.



4. Compare your little cubes to the original. You might start by putting a small cube on the outline of the base of the original cube as shown in the illustration.



5. Look at the chart below. The first line describes the big cube. You can describe the smaller cubes in terms of the big cube. This will most often be a fractional amount. For example, if you look at your little cube on the outline of the big cube, you can determine what fraction of the big cube base area the base of the little cube is. Now fill in the second line of the chart.

Length of Side per Cube	Area of Base per Cube	Total Surface Area per Cube	Total Number of Cubes	Volume per Cube	Total Surface Area of All Cubes
1	1	6	1	1	6
½					
⅓					

6. Rebuild your large cube, then cut it to create little cubes that are a third the length of the large cube per side. Fill in the third line of the chart.

7. What happens to length, surface area, and volume as a cube shrinks? Do you notice any patterns?

What's Going On?

As the length of a side decreases, surface area and volume decrease as well, but they decrease by different amounts. If the length of a side is halved, the area is ¼ of the original area. This is true for all areas: area of the base, total surface area, and area of a cross section. In general, if you make a side of a cube $1/n$ long, then the areas are $1/n \times 1/n$, or $1/n^2$ of the original areas.

To find the total surface area of all the little cubes relative to the big cube, multiply the surface area of one cube by the total number of cubes. For a cube that is ½ the original length, the total surface area is ¼ × 6, or 6⁄4. So the surface area of all the cubes is 6⁄4 × 8, or 12—twice the surface area of the big cube. *You have much more surface area for a fixed volume.*

When the length of a side is halved, the volume is ⅛ of the original volume. If you make a side of a cube $1/n$ long, then the volume is $1/n \times 1/n \times 1/n$, or $1/n^3$. Small things have a high surface-to-volume ratio—that is, they have a lot of surface area relative to their volume. For large things, the opposite is true.

Were these your answers?

Length of Side per Cube	Area of Base per Cube	Total Surface Area per Cube	Total Number of Cubes	Volume per Cube	Total Surface Area of All Cubes
1	1	6	1	1	6
½	¼	6⁄4	8	⅛	12
⅓	1⁄9	6⁄9 = ⅔	27	1⁄27	18

Going Further

Now that you know what the rules are, see if you can figure out mathematically what will happen to the area and volume if you decrease the side of a cube to ¼ and ⅕ of the original length. (If you'd rather, you could rebuild and cut up your cube again.)

Length of Side per Cube	Area of Base per Cube	Total Surface Area per Cube	Total Number of Cubes	Volume per Cube	Total Surface Area of All Cubes
1	1	6	1	1	6
¼					
⅕					

What's the Nanoscale Connection?

Nanoparticles have an *enormous* amount of surface area compared to their volume. Imagine that you have a plasticine cube 1 centimeter per side; its total surface area would be 6 cm^2—about the size of a business card. Now imagine that you could cut your cube into tiny cubes that measure 1 nanometer per side. A nanometer-long cube would be 1/10,000,000 the length of the centimeter-sized cube, and the total surface area of all the nano-sized cubes would be 60,000,000 cm^2 or 6,000 m^2—larger than a football field!

Because of the huge amount of surface area compared to the volume, nanoparticles tend to have different properties (including mechanical, electronic, and chemical properties) than larger amounts of the same material. Scientists and engineers are taking advantage of these different properties to design new products.

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